

TMC - 5



TeenEagle
MATHS

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Qualification Round Questions

1. (Logical Thinking)

3 points

A mosaic is 74 years older than the engraving. Thirty years ago, the mosaic was three times as old as the engraving. Find the present age of each.

- (A) Mosaic: 141 Engraving: 67
- (B) Mosaic: 137 Engraving: 51
- (C) Mosaic: 151 Engraving: 57
- (D) Mosaic: 127 Engraving: 87

2. (Logical Thinking)

3 points

Given A and C are two non-zero digits and the 3-digit numbers formed by these two digits have the following properties:

- (a) CAA is divisible by 9;
- (b) ACA is divisible by 7;
- (c) AAC has an odd number of factors.

Find the 3-digit number CAA .

- (A) 488
- (B) 499
- (C) 522
- (D) 544

3. (Logical Thinking)

3 points

Split 2, 6, 8, 9, 11, 14, 15 and 17 into 2 groups of 4 numbers such that the sums of numbers in both groups are equal.

Find the product of the numbers in the group that contains 17.

- (A) 224
- (B) 476
- (C) 1904
- (D) 3808

4. (Logical Thinking) 3 points

There are 6 pieces of white chopsticks, 3 pieces of yellow chopsticks and 4 pieces of brown chopsticks mixed together.

Close your eyes.

If you want to get one pair of each color (white, yellow and brown chopsticks) at least how many pieces of chopsticks need to be taken?

- (A) 14 (B) 12 (C) 11 (D) 10

5. (Logical Thinking) 3 points

30 children, numbered 1 to 30, sit around a straight line in order.

Each child has an integer in hand. The child numbered 1 has the integer 1, the child numbered 2 has the integer 2, the child numbered 3 has the integer 3.

Given that the sum of the integers of any 6 consecutive children is equal to 40.

What is the product of integers of children numbered 14 and children numbered 27?

- (A) 3 (B) 4 (C) 5 (D) 6

6. (Logical Thinking) 3 points

Use digits 5, 6, 7, 8 and 9 to form 4-digit numbers (Digits cannot be repeated).

How many of them are divisible by 11?

- (A) 24 (B) 18 (C) 16 (D) 8

7. (Logical Thinking) 3 points

There are 25 problems in a mathematics competition. The scores for each problem are allocated in the following ways: 2 marks will be given for a correct answer. Only 1 mark will be deducted for a wrong answer and 0 marks will be given for a blank answer.

Find the minimum number of candidate(s) to ensure that 2 candidates will have the same score in the competition.

- (A) 64 (B) 68 (C) 76 (D) 84

8. (Logical Thinking)

3 points

30000 children, numbered 1 to 30000, sit around a circle in order. Each child has an integer in hand. The child numbered 1 has the integer 2. Given that the sum of the integers of any 2017 consecutive children is equal to 4034.

What is the integer held by the child numbered 24601?

- (A) 2 (B) 4 (C) 6 (D) 8

9. (Logical Thinking)

3 points

Use numbers 1 to 5 to fill in the boxes in the table below such that all five numbers in every row, every column and the 2 diagonals are different.

What number should be written in the box X?

- (A) 5 (B) 3 (C) 2 (D) 1

1	2	3	4	
				X
	3			1
5			3	
		5		

10. (Logical Thinking: Speed-distance-time problems)

3 points

There are 100 students attending a mathematics contest. Among the 10 problems in the mathematics competition, the numbers of correct answers for each question are 81, 72, 95, 80, 81, 65, 76, 70, 78 and 62 respectively.

A student is awarded if he/she attains 7 or more correct answers.
At least how many students are awarded?

- (A) 64 (B) 56 (C) 48 (D) 40

11. (Functions)

4 points

The function $g(x) = \frac{4}{5x+2}$ for $x \geq 0$.

Find the domain of $g^{-1}(x)$.

- (A) $0 < x \leq 2$ (B) $-2 < x < 2$ (C) $-2 < x < 0$ (D) $-2 \leq x \leq 2$

12. (Functions)

4 points

The function f is defined by $f: x \mapsto 4\sin x - 1$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$.

Find the coordinates of the points at which the curve $y=f(x)$ intersects the coordinates axes.

- (A) $x = 0$ only
- (B) $x = 0$ and $x = 0.253$
- (C) $x = -0.253$ only
- (D) No real solutions

13. (Coordinate geometry)

4 points

The equation of a curve $y = 2x + \frac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

Find the set of values of k for which the line does not meet the curve.

- (A) $-8 \leq k \leq 8$
- (B) $-12 < k < 12$
- (C) $-3 < k < 12$
- (D) $-12 \leq k < 12$

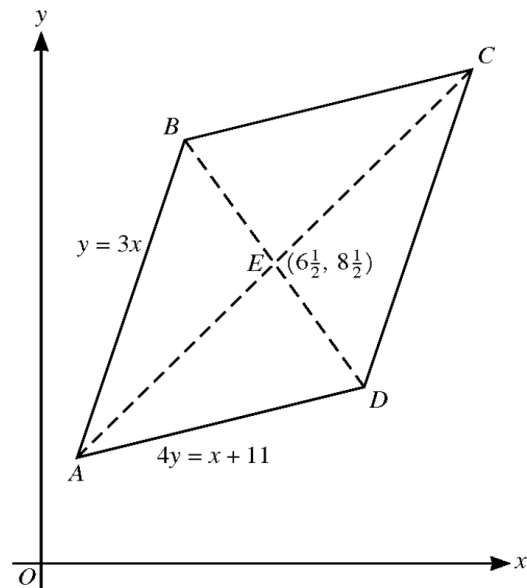
14. (Coordinate geometry)

4 points

The diagram shows a parallelogram $ABCD$, in which the equation of AB is $y=3x$ and the equation of AD is $4y=x+11$. The diagonals AC and BD meet at the point $E\left(6\frac{1}{2}, 8\frac{1}{2}\right)$.

Find, by calculation, the coordinates of A , B , C and D .

- (A) $A(1, 3)$, $B(4, 12)$, $C(12, 14)$, $D(9, 5)$
- (B) $A(3, 1)$, $B(12, 4)$, $C(14, 12)$, $D(5, 9)$
- (C) $A(4, 12)$, $B(1, 3)$, $C(5, 9)$, $D(12, 14)$
- (D) $A(9, 5)$, $B(4, 12)$, $C(1, 3)$, $D(14, 12)$



15. (Functions)

4 points

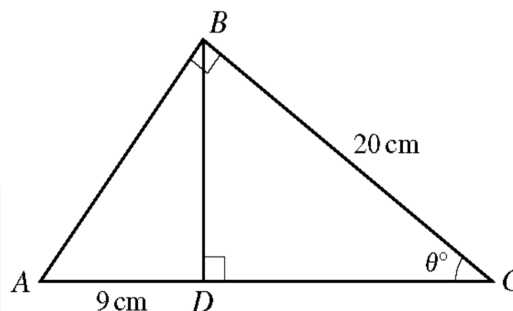
If a is a real number, determine the maximum value of $3a - a^2 + 16$.

- (A) $-\frac{3}{2}$ (B) $\frac{53}{2}$ (C) $\frac{73}{4}$ (D) $-\frac{73}{4}$

16. (Trigonometry)

4 points

The diagram shows a triangle ABC in which $BC = 20 \text{ cm}$ and $\angle ABC = 90^\circ$. The perpendicular from B to AC meets AC at D and $AD = 9 \text{ cm}$. Let $\angle BCA = \theta$.



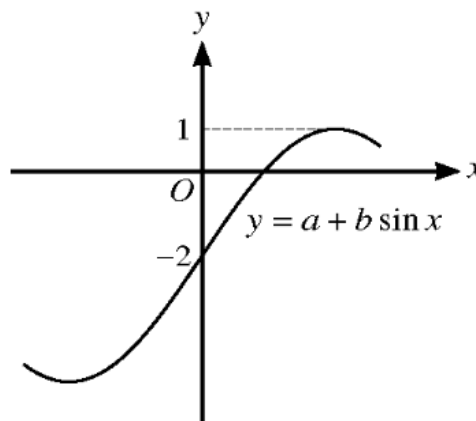
Find the exact value of $\cos\theta$.

- (A) $\cos\theta = \frac{1}{2}$ (B) $\cos\theta = \frac{4}{5}$ (C) $\cos\theta = 0.6$ (D) $\cos\theta = 0.7$

17. (Trigonometry)

4 points

The diagram shows part of the graph of $y = a + b \sin x$.



Find the values of the constants a and b .

- (A) $a = 2, b = -3$
 (B) $a = 3, b = 2$
 (C) $a = -2, b = -3$
 (D) $a = -2, b = 3$

18. (Exponential & Logarithmic equations)

4 points

Find the value of x for which $3(2^{1-x}) = 7^x$.

Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers.

- (A) $\frac{\ln(6)}{\ln(14)}$ (B) $\frac{\ln(3)}{\ln(7)}$ (C) $\frac{\ln(14)}{\ln(6)}$ (D) $\frac{\ln(7)}{\ln(3)}$

19. (Integration)

4 points

Find the value of $\int_0^1 (2 - x)e^{-2x} dx$.

- (A) $\frac{3}{4}(1 - e^2)$
- (B) $\frac{1}{3}(3e - e^{-2})$
- (C) $\frac{1}{4}(3 - e^{-2})$
- (D) $\frac{1}{3}(1 - e^2)$

20. (Series)

4 points

The first three terms of an arithmetic progression are $2\sin x$, $3\cos x$ and $(\sin x + 2\cos x)$ respectively, where x is an acute angle.

Find the exact value of $\tan x$.

- (A) $\tan x = \frac{4}{3}$
- (B) $\tan x = \frac{3}{4}$
- (C) $\tan x = -\frac{4}{3}$
- (D) $\tan x = -\frac{3}{4}$

21. (Differentiation)

4 points

The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function.

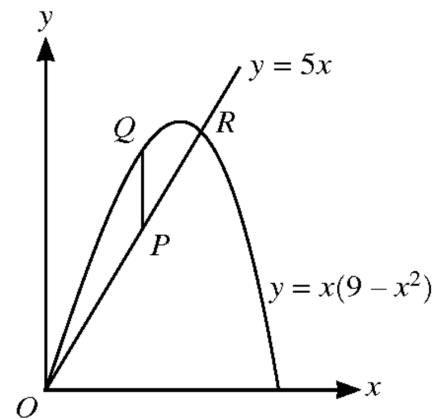
Find the least possible value of n .

- (A) $n < 3$
- (B) $n \geq 2$
- (C) $n > 3$
- (D) $n = 3$

22. (Pure mathematics: Differentiation)

5 points

The diagram shows part of the curve $y = x(9 - x^2)$ and the line $y = 5x$, intersecting at the origin O and the point R . Point P lies on the line $y = 5x$ between O and R , and the x -coordinate of P is t . Point Q lies on the curve and PQ is parallel to the y -axis.



Given that t can vary, find the maximum value of the length of PQ .

- (A) $\frac{16\sqrt{3}}{9}$ (B) $\frac{16}{\sqrt{2}}$ (C) $\frac{16\sqrt{3}}{3}$ (D) $\frac{16}{\sqrt{18}}$

23. (Mechanics: Kinematics)

5 points

A particle P moves in a straight line, starting from the rest at a point O on the line.

At time t s after leaving O , the acceleration of P is $k(16 - t^2) \text{ m s}^{-2}$, where k is a positive constant, and the displacement from O is s m.

The velocity of P is 8 m s^{-1} when $t = 4$.

Find the speed of P at the instant that it returns to O .

- (A) 12.46 m/s^2 (B) 18.4 m/s^2 (C) 29.4 m/s^{-1} (D) 32.24 m/s^{-1}

24. (Statistics and Probability: Coded data)

5 points

For n values of the variable x , it is given that

$$\sum(x - 200) = 446, \quad \sum x = 6846.$$

Find the value of n .

- (A) 32 (B) 46 (C) 58 (D) 62

25. (Statistics and Probability: Permutations and Combinations)

5 points

In how many different ways can the 9 letters of the word TELESCOPE be arranged so that there are exactly two letters between the letters T and C?

- (A) 10 460 (B) 10 560 (C) 15 080 (D) 10 080

26. (Statistics and Probability: Permutations and Combinations)

5 points

A bag contains 12 marbles, each of a different size. Eight of the marbles are red and four are blue.

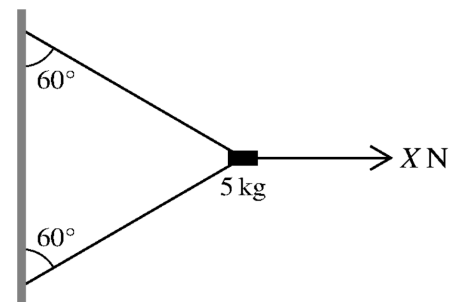
How many selections of 5 marbles contain at least 4 marbles of the same colour?

- (A) 540 (B) 344 (C) 280 (D) 56

27. (Mechanics: Forces and Equilibrium)

5 points

A block of mass 5 kg is held in equilibrium near a vertical wall by two light strings and horizontal force of magnitude X N, as shown in the diagram. The two strings are both inclined at 60° to the vertical.



Given that $X = 100$, find the tension in the lower string.

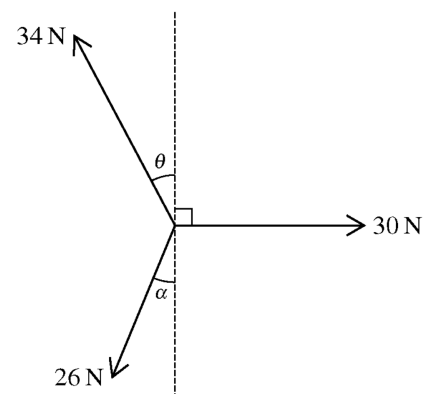
- (A) $T_L = 10$ N (B) $T_L = 8.54$ N (C) $T_L = 7.2$ N (D) $T_L = 7.74$ N

28. (Mechanics: Forces and Equilibrium)

5 points

Coplanar forces of magnitudes 34 N, 30 N and 26 N act at a point in the directions shown in the diagram.

Given that $\sin \alpha = \frac{5}{13}$ and $\sin \theta = \frac{8}{17}$, find the magnitude and direction of the resultant of the three forces.



- (A) $R = 27$ N (B) $R = \sqrt{52}$ N (C) $R = 3\sqrt{3}$ N (D) $R = 30$ N

29. (Mechanics: Momentum)

5 points

Three particles P , Q and R of masses 0.1 kg , 0.2 kg and 0.5 kg respectively, are at rest in a straight line on a smooth horizontal plane.

Particle P is projected towards Q at a speed of 5 m s^{-1} . After P and Q collide, P rebounds with speed 1 m s^{-1} .

Find the speed of Q immediately after the collision with P .

- (A) 5 m s^{-1} (B) 4 m s^{-1} (C) 3 m s^{-1} (D) 2 m s^{-1}

30. (Mechanics: Momentum)

5 points

Two small smooth spheres A and B , of equal radii and masses of 4 kg and $m \text{ kg}$ respectively, lie on a smooth horizontal plane. Initially, sphere B is at rest and A is moving towards B with speed 6 m s^{-1} .

After the collision, A moves with the speed of 1.5 m s^{-1} and B moves with the speed of 3 m s^{-1} .

Find the two possible values of the loss of kinetic energy due to the collision.

- (A) loss of $KE = 40.5 \text{ J}$ or 22.5 J
 (B) loss of $KE = 36 \text{ J}$ or 25 J
 (C) loss of $KE = 22.25 \text{ J}$ or 42.25 J
 (D) loss of $KE = 20.5 \text{ J}$ or 40.5 J

Answer Key

1	A	11	A	21	D
2	C	12	B	22	A
3	D	13	B	23	C
4	B	14	A	24	A
5	D	15	C	25	D
6	A	16	B	26	B
7	C	17	D	27	D
8	A	18	A	28	B
9	B	19	C	29	C
10	D	20	A	30	A